MATH 105 101 Midterm 1 Sample 5

1. (15 marks)

   (a) (4 marks) Given the function:

   $$f(x, y) = x \sin y,$$

   find $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y^2}$ at the point $(2, \pi/6)$. *Simplify your answers.*

   (b) (3 marks) Find all values of a such that the point $(a, 1)$ lies on the level curve of the function $z = f(x, y) = x^2 + xy$ for $z_0 = 6$.

   (c) (2 marks) Let $\mathbf{v} = \langle \frac{3}{4}, -\frac{1}{2} \rangle$, and $\mathbf{w} = \langle 16, -6 \rangle$. Determine if $\mathbf{v}$ is perpendicular to $\mathbf{w}$. Justify your answer.

   (d) (3 marks) A plane $\mathcal{P}$ is parallel to the plane $2x + z = y$ and passes through the point $P(1, -1, 3)$. Find the equation of the plane $\mathcal{P}$.

   (e) (3 marks) Given functions:

   $$F(x, y) = x + e^y, \qquad G(x, y) = y + e^x,$$

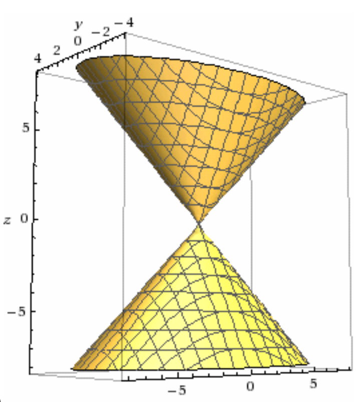
   does there exist a function $f(x, y)$ such that $\nabla f(x, y) = \langle F, G \rangle$? Justify your answer and clearly state any result that you may use.

2. (5 marks) Consider the surface S given by:

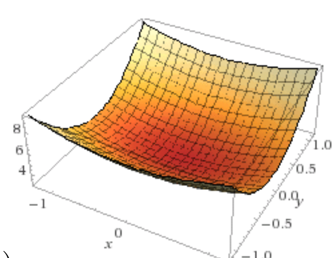
   $$z = 3 + x^2 + 4y^2.$$

   (a) (4 marks) Find and sketch the traces of S in the $z = 4$ and $x = 0$ planes.

   (b) (1 mark) Based on the traces you sketched above, which of the following renderings represents the graph of the surface?



(*A*)



(*B*)

3. (10 marks) Let R be the triangle whose vertices are $(2, 0)$, $(-1, 0)$ and $(0, 2)$. Find the maximum and minimum values of the function

   $$f(x, y) = x^2 - 2x + y.$$

   on the *boundary of the region R*.

4. (10 marks) Find *all* critical points of the following function:

$$f(x, y) = e^{-\frac{1}{3}x^3+x-y^2}.$$

Classify each point as a local minimum, local maximum, or saddle point. *You do not have to solve for extrema on the boundary.*

5. (10 marks) A candy company produces boxes of bubblegum and gummy bears. It costs the company \$1 to produce a box of either type of candy. On the other hand, bubblegum sells for \$3 per box and gummy bear for \$5 per box. Due to limitations of sugar supply, the production scheme has to satisfy the production possibilities curve:

$$\sqrt{x} + 2\sqrt{y} = 300,$$

where x and y denote the number of boxes of bubblegum and gummy bears that the company produces weekly. Assuming that the company manages to sell every unit produced, use the method of Lagrange multipliers to answer the following question: how many boxes of each type of candy should the company aim to produce if it is to maximize profit?

Clearly state the objective function and the constraint. *You are not required to justify that the solution you obtained is the absolute maximum.* **A solution that does not use the method of Lagrange multipliers will receive no credit, even if it is correct.**